Solitons in a Three-Wave System with Intrinsic Linear Mixing and Walkoff

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Abstract

A modification of the usual Type-II second-harmonic-generation model is proposed, which includes two additional features: linear conversion and walkoff (group-velocity difference) between two components of the fundamental-frequency (FF) wave. Physical interpretations of the model are possible in both temporal and spatial domains. In the absence of the intrinsic walkoff, the linear mixing makes real soliton solutions stable or unstable, depending on the relative sign of the two FF components. Unstable solitons spontaneously re-arrange themselves into stable ones. Fundamental solitons change their shape (in particular, they develop chirp) but remain stable if the intrinsic walkoff is included. In addition, quasi-stable double-humped solitary waves are found in the latter case.

1 Introduction and formulation of the model

Spatial and temporal solitons in various models with quadratic ($\chi^{(2)}$)) nonlinearity, including the second-harmonic-generating (SHG) and more general three-wave (3W) systems, have attracted a lot of attention, see reviews [1, 3, 4] and a special volume [5]. The simplest (degenerate) SHG model involves only two waves, the fundamental-frequency (FF) one and its second harmonic (SH). In a general situation, one is dealing with a 3W system, that involves two "daughter" waves and a pump; it is known that the 3W system also gives rise to soliton solutions [6]. A case of great practical interest is a more special version of the 3W system, known as the Type-II SHG model, in which both "daughters" represent two polarizations of the FF wave, while SH has a single polarization. The latter model corresponds to the most typical experimental conditions [1], and its three-component soliton solutions (also called vectorial solitons) were studied in detail [7, 8, 2, 3].

Various modifications of the Type-II system were introduced, and the soliton solutions in them were studied [see, e.g., Refs. [9]]. These modifications were focused on adding extra quadratic terms to the model's equations. Another possibility which has not yet been considered, except for Ref. [10] (see below), is to introduce linear coupling (mixing) between the two FF components. The simplest physical realization of this possibility may be found in terms of the Type-II SHG process in a fiber-like birefringent waveguide, which is subjected to a twist that mixes the two FF polarizations. Note that twist-induced linear mixing between two linear polarizations is a well-known feature of optical fibers with the Kerr ($\chi^{(3)}$) nonlinearity [11]. Of course, application of the twist to a monocrystalline waveguide in which SHG takes place is problematic, but an effective twist, without applying any mechanical torque, can be created in an evident way in the practically important case when SHG itself is induced by means of periodic poling of the host medium (see a review [12]).

A system of equations to describe the Type-II SHG in a birefringent medium in the presence of the linear mixing can be derived as a straightforward generalization of the model developed in Ref. [8]:

$$
\begin{aligned}
i\delta A_z + ib_1 A_\tau + (1/2)\,A_{\tau\tau} - \beta A + B^* C &= \kappa B, &(1)\\
i\delta^{-1} B_z + ib_2 B_\tau + (1/2)\,B_{\tau\tau} - \beta^{-1} B + A^* C &= \kappa A, &(2)\\
i\sigma C_z + (1/2)\,D C_{\tau\tau} - \alpha C + AB &= 0, &(3)
\end{aligned}
$$

where the asterisk and subscripts stand, respectively, for the complex conjugation and partial derivatives, the evolution in the temporal domain is implied, τ being the standard reduced-time variable [13], complex fields $A(z,\tau)$, $B(z,\tau)$ and $C(z,\tau)$ represent the two FF components and SH wave, respectively, β is a phase-birefringence parameter, α measures the phase mismatch between the FF and SH waves, $\delta > 0$ is another parameter taking into regard asymmetry between the two FF components [8], D is a relative SH/FF dispersion coefficient, and $\sigma > 0$ determines a relative propagation constant, all these parameters being real.

New (in comparison with the known model) terms introduced in Eqs. (1) and (2) are the linear-mixing ones with the real coefficient κ, and group-velocity-birefringence (temporal-walkoff) terms with real coefficients $b_{1,2}$ [the reference frame is defined so that the group-velocity term vanishes in Eq. (3)]. Note that in the absence of the linear mixing, $\kappa = 0$, the walkoff terms can be easily removed from the model by means of phase transformations and change of the reference frame, therefore the model elaborated in Ref. [8] did not include these terms. However, these terms are irreduceable if $\kappa \neq 0$.

The model may also be interpreted in the spatial domain, with τ is replaced by the transverse coordinate x, $b_{1,2}$ being spatial walkoff [14] parameters. In fact, the linear coupling between the FF components in the 3W model in the spatial domain (a planar optical waveguide with the $\chi^{(2)}$ nonlinearity) may be induced in a simple way by means of a Bragg grating (a system of parallel scores) written on the waveguide [10]. In the latter case, all the fields have the same polarization, the difference between the two FF components being in the direction of their Poynting vectors.

An objective of this work is to find fundamental-soliton solutions of the model and test their stability by means of precise numerical simulations. Prior to that, we will consider the model's linear spectrum, which is necessary in order to realize what type of solitons may exist in it.

For $\kappa \neq 0$ but $b_{1,2} = 0$, stationary soliton solutions to Eqs. (1), (2), and (3) can be easily found. A dynamical test will show that the stationary solitons may be both stable and unstable in this case, depending on the relative sign of the two FF components, which is a difference from the results reported in Ref. [8] for the Type-II SHG model without the linear mixing ($\kappa = 0$). With the introduction of the walkoff parameters ($b_{1,2} \neq 0$), numerical solution of the stationary equations becomes difficult, therefore we rely upon direct simulations in this case: propagation of an initial solitary-wave configuration, which is taken as a stable soliton for the same values of parameters but with $b_1 = b_2 = 0$, generates a stable fundamental soliton, featuring intrinsic chirp, after a transient process. Additionally, we also find quasi-stable double-humped solitary waves in the latter case.

2 The linear spectrum

Solitons are represented by solutions to Eqs. (1) - (3) of the form

$$A(z,\tau) = e^{iKz}a(\tau),\ B(z,\tau) = e^{iKz}b(\tau),\ C(z,\tau) = e^{2iKz}c(\tau), \tag{4}$$

where K is a real propagation constant, and $a(\tau)$, $b(\tau)$, and $c(\tau)$ are even functions (complex ones, in the general case) that must vanish at $|\tau| \to \infty$. As is commonly known, (bright) soliton solutions may only exist at values of K that do not overlap with the continuous spectrum of the FF part of the system. The spectrum is determined by the substitution of the continuous-wave solution,

$$A(z,\tau) = A_0 \exp\left(iKz - i\omega\tau\right), B(z,\tau) = B_0 \exp\left(iKz - i\omega\tau\right), \tag{5}$$

into linearized versions of Eqs. (1) and (2).

As for the SH component, a similar condition for the existence of solitons (the non-overlapping with the continuous spectrum) is not always necessary in it, since situations are possible when the SH equation may *not* be linearized, its quadratic term being everywhere (at all values of τ, including $|\tau| \to \infty$) on the same order of magnitude as the linear terms. Such a situation is possible as the quadratic and linear terms in the SH equation are composed of different fields: the former one contains only the FF components, while the linear terms are expressed in terms of the SH field, see Eq. (3). Eventually, this leads to a possibility of the existence of the so-called embedded solitons, for which the SH propagation constant $2K$ [see Eq. (4)] indeed falls into the linear spectrum of the SH wave [18, 19].

The combination of the linear coupling and group-velocity difference between the two FF components in Eqs. (1) and (2) makes the present system somewhat akin to $\chi^{(2)}$ models in which effective dispersion and/or diffraction are induced by a Bragg grating, while the intrinsic dispersion (or diffraction) are neglected. Models of this type, which have a finite gap in their linear spectrum and, accordingly, give rise to *gap solitons* supported by the quadratic nonlinearity, have attracted considerable attention, see Refs. [15, 16, 17, 10]. However, the present model, despite its similarity to the gap-soliton ones, has the linear spectrum of a different type, which contains no finite gap but, instead, the usual semi-infinite gap extending to $K \to +\infty$. Indeed, taking, in order to avoid ponderous formulas, Eqs. (1) and (2) with $\delta = 1$, $\beta = 1$, $b_1 = -b_2 \equiv b$, and substituting the expressions (5) into the linearized equations, one can easily obtain the following expression for the linear spectrum:

$$K = -\left(1 + \omega^2/2\right) \pm \sqrt{(b\omega)^2 + \kappa^2}. \tag{6}$$

It is straightforward to see that the combination of two branches of this spectrum does not yield any finite gap [that would exist, in the form $(K+1)^2 < \kappa^2$, if the dispersion terms in Eqs. (1) and (2), and hence the term $\omega^2/2$ in Eq. (6), were omitted]. On the other hand, all sufficiently large positive values of K do not belong to the linear spectrum, forming the above-mentioned semi-infinite gap (in the case $\kappa^2 > b^4$, it simply takes the form $K > 0$).

Thus, the present system combines, to a certain extent, previously studied ordinary soliton models and those which give rise to gap solitons, which makes it interesting to search for solitons in it.

3 Solitons in the model without intrinsic walkoff

3.1 Unstable solitons

We start by seeking for soliton solutions to Eqs. (1) - (3) in the case $b_{1,2} = 0$ (no intrinsic walkoff, while the linear mixing is present, $\kappa \neq 0$). The stationary version ($\partial/\partial z = 0$) of the system of Eqs. (1), (2) and (3) then becomes real, and, accordingly, stationary solutions are looked for in a real form by means of

the shooting method. Note that, in the absence of the linear mixing ($\kappa = 0$), solutions differing by a relative sign of the FF components, A and B, are, obviously, equivalent. However, this is not the case if $\kappa \neq 0$ (which is similar to the situation in a $\chi^{(2)}$ model of a dual-core waveguide, where linear terms couple FF fields in two cores [20]). In particular, it is obvious that Eqs. (1) – (3) can be derived from a Hamiltonian, its term which accounts for the linear mixing being

$$H_{\mathrm{mix}} = \kappa \int_{-\infty}^{+\infty} \left(A^* B + A B^*\right) d\tau \,. \tag{7}$$

In the case of real solutions, and taking, by definition, $\kappa > 0$, one may expect that the solutions with $\mathrm{sgn}A = \mathrm{sgn}B$ should be unstable, as they make the term (7) of the Hamiltonian positive, while solutions with $\mathrm{sgn}A = -\mathrm{sgn}B$ may be stable, as they yield $H_{\mathrm{mix}} < 0$ (a known principle states that a solution which makes the value of the Hamiltonian larger is likely to be unstable [21]).

The application of the shooting method to the stationary real equations (1) - (3) with $b_{1,2} = 0$ indeed yields stationary solitons in a broad parametric region, provided that the relative dispersion coefficient is positive, $D > 0$. A typical example of a solution with $\mathrm{sgn}A = \mathrm{sgn}B$, generated by the shooting method, is displayed in Fig. 1.

Direct simulations of the dynamical stability of the thus found stationary solitons were performed by means of the implicit Crank-Nicholson scheme. As is known [22], this scheme has advantages over the split-step method for very long beam-propagation simulations. To control the accuracy of the direct simulations, we made use of the fact that the underlying system (1) – (3) conserves the net energy of the three fields,

$$E = \int_{-\infty}^{+\infty} \left(\delta \left|A\right|^2 + \delta^{-1} \left|B\right|^2 + \sigma \left|C\right|^2\right) d\tau \,. \tag{8}$$

Results of the simulations complied with the conservation of the integral (8) to a very high accuracy.

The application of the Crank-Nicholson scheme to the soliton shown in Fig. 1 has produced a picture which is displayed, in the contour-plot form, in Fig. 2. This result, as well as many other runs of simulations for other stationary solitons, demonstrate that, in accordance with the general argument presented above, all the real solitons with $\mathrm{sgn}A = \mathrm{sgn}B$ are unstable. It is relevant to compare a characteristic propagation length z_{stab} before the onset of the instability, which is $\simeq 80$ in Fig. 2, and the soliton period for the same pulse (which is defined as the propagation length necessary for the change of the internal phase of the soliton by $\pi/2$ [13]), that can be estimated for the soliton shown in Fig. 1, in terms of its width W, as $z_{\mathrm{sol}} \sim \pi W^2 \simeq 15$. This means that the unstable soliton can pass ~ 5 soliton periods as a quasi-stable object. In a typical experimental situation for spatial solitons, their diffraction length is ~ 1 mm, while the size of a sample is a few cm [1], hence both the unstable soliton itself and its instability may be experimentally observed in the spatial domain.

In fact, Fig. 2 displays a typical scenario of the development of the instability of solitons with sgn$A$ = sgn$B$: after a relatively long period of a very slow "latent" growth of the instability, an abrupt explosion occurs, as a result of which the soliton sheds off some amount of radiation in the SH component, and rearranges itself into a new, completely stable, soliton (these stable solitons will be described below). Figure 3 shows the quasi-stable-propagation distance $z_{\rm stab}$ of the unstable solitons vs. the coupling constant κ, for several different values of the relative SH dispersion coefficient D. The fact that, irrespective of the value of D, the distance $z_{\rm stab}$ diverges as $\kappa \to 0$ is quite natural, as in this limit we get back to the usual model without the linear mixing, where the real solitons are stable irrespective of the relative sign of their two FF components [8].

If SH dispersion coefficient D is very small, the character of the instability development becomes qualitatively different from that illustrated by Fig. 2: as is seen in Fig. 4, in this case only the SH component of the soliton survives. Note that, in the case $D = 0$, Eqs. (1), (2), and (3) have an obvious solution corresponding to the eventual state observed in Fig. 4: $A = B = 0$, $C(z,\tau) = \exp\left(-i\alpha\sigma^{-1}z\right) \cdot c(\tau)$, where $c(\tau)$ is an arbitrary function.

3.2 Stable solitons

In the case $b_1 = b_2 = 0$, another class of real stationary soliton solutions can be found (by means of the shooting method too), with opposite signs of the two FF components (sgn$A$ = $-$sgn$B$). A typical example of such a soliton is shown in Fig. 5. In accordance with the qualitative arguments given above [based on the sign of the coupling term (7) in the Hamiltonian], the solitons of this type are found to be completely stable in direct simulations of their evolution. An example is displayed in terms of the contour plots in Fig. 6 (note that the full propagation distance presented in Fig. 6 is ~ 100 soliton periods; in fact, the stability was seen, in much longer simulations, to persist indefinitely).

An accurate analysis of the stable pulses, the formation of which was observed as a result of the development of the instability of the stationary solitons with sgn$A$ = +sgn$B$ (see Fig. 2), shows that the appearing stable pulses are identical to the stable solitons of the type considered here, with sgn$A$ = $-$sgn$B$. Thus, these solitons are really robust, playing a role of *attractors* in the evolution of unstable pulses (the existence of effective attractors in a conservative nonlinear-wave system is possible due to radiation losses).

4 Solitons in the system with the intrinsic walkoff

4.1 Single-humped solitons

It is quite interesting to understand how solitons are modified if the group-velocity (walkoff) terms are restored in Eqs. (1) and (2). Search for stationary soliton solutions of the full system of Eqs. (1) - (3), which include the walkoff

terms, turns out to be much harder than in the case $b_1 = b_2 = 0$, as it appears quite difficult to secure convergence of results produced by the shooting method. Therefore, we adopted an approach based on direct simulations of the full equations in the following fashion: stationary solutions corresponding to stable solitons (with $\mathrm{sgn}A = -\mathrm{sgn}B$), which were found above for the case $b_1 = b_2 = 0$ (for instance, the soliton shown in Fig. 5), were used as initial conditions for simulations of the evolution equations (1) − (2) with the same values of all the parameters but b_1 and b_2. Note that the introduction of the walkoff terms must essentially rearrange the input pulses, as they are real, while stationary solutions to Eqs. (1) − (2) with $b_{1,2} \neq 0$ cannot be real.

A typical example of the evolution of the thus chosen input pulse is shown, by means of contour plots, in Fig. 7 [in this figure, we display the evolution of local powers $|A(\tau)|^2$, $|B(\tau)|^2$, and $|C(\tau)|^2$]. The propagation distance in Fig. 7 is extremely large ($\simeq 200$ soliton periods), which was taken in order to make it sure that a final soliton, if any, takes a sufficiently well-established form. The result of the evolution is shown in Fig. 8.

A conclusion suggested by this and many other runs of simulations is that stable solitons with intrinsic chirp establish themselves in the presence of the intrinsic walkoff, although radiation shed off from the soliton in the course of its self-adjustment separates from it very slowly. The latter peculiarity can be understood. Indeed, the radiation tail attached to the soliton in Fig. 8 (upper panel) is all built of the SH field, which has zero group velocity in the underlying equation (3), hence it does not readily separate from the zero-velocity soliton. It seems very plausible (although detailed consideration of the issue is beyond the scope of this work) that Eqs. (1) − (3) may also generate moving ("walking") solitons, in which case the separation of the soliton from the radiation "garbage" would probably be faster.

To further check that the (quasi-) soliton (called this way because of the radiation tail attached to it), whose formation and structure are shown in Figs. 7 and 8, has long since completed any essential evolution, in Fig. 9 we show the evolution (vs. z) of the net energy (or intensity, in the case of beams in the spatial domain) of each component of the soliton, i.e.,

$$\int_{-\infty}^{+\infty} |A(z,\tau)|^2 \, d\tau, \ \int_{-\infty}^{+\infty} |B(z,\tau)|^2 \, d\tau, \ \int_{-\infty}^{+\infty} |C(z,\tau)|^2 \, d\tau \tag{9}$$

($\int_{-\infty}^{+\infty}$ is realized as the integral over the whole simulation domain), and Fig. 10 displays the evolution of the total energy defined by Eq. (8). A very small initial loss of the total energy (see Fig. 10) is explained by leakage across borders of the integration domain in the process of the initial rearrangement of the soliton. Note that intensive energy exchange between the A and C fields (see Fig. 9) is limited to approximately the same initial stage of the evolution at which the energy loss takes place; then, any tangible evolution ceases, in terms of the integral field characteristics (9).

4.2 Double-humped structures

The approach described above produces stable *fundamental* solitons, i.e., single-humped ones. On the other hand, it is well known that $\chi^{(2)}$ models readily give rise to higher-order solitons – first of all, double-humped ones [23] – which, however, are always unstable in standard models, including the 3W Type-II model [3]. Search for *stable* double-humped solitons in various systems is a problem of considerable interest for physical applications, see, e.g., Refs. [24]. In fact, the first examples of (numerically) stable one-dimensional double-humped solitons were found in 3W models combining $\chi^{(2)}$ nonlinearity and linear coupling (which was induced by the Bragg grating) between two components of the FF field [10, 17]. Moreover, the model introduced in Ref. [10], that seems to be closest to the one considered in the present work, gives rise also to vast families of double- and multi-humped embedded solitons [19], although the stability of those solutions was not studied in detail.

We made an attempt to search for double-humped solitary-wave structures in the present model. In the absence of the linear mixing ($b_{1,2} = 0$), they have never been found, which seems quite natural in view of the above-mentioned results obtained in allied models. However, structures of that type can indeed be found at finite (actually, quite small) values of $|b_{1,2}|$, and they seem to be nearly stable, although they are not completely stationary.

To this end, the stationary version of Eqs. (1) - (3), produced by the substitution of the waveforms (4), was first solved numerically with high but finite accuracy, starting from the numerically exact solution for $b_1 = b_2 = 0$, such as the one shown in Fig. 5, and gradually increasing the parameter $b_1 = -b_2 \equiv b$. As it was mentioned above, in the presence of the walkoff terms with $b_{1,2} \neq 0$ straightforward application of the shooting technique does not provide for convergence of soliton solutions to indefinitely high accuracy, this is why the accuracy was finite, as mentioned above. It was observed that if other parameters keep constant values (for instance those which are mentioned in the caption to Fig. 11), the increase of b makes the (finite-accuracy) soliton broader, and *splitting* of the soliton's crest into two takes place at $b = 0.00274$. A typical example of the appearing double-humped structure is shown, for a slightly larger value $b = 0.003$, in the upper panel of Fig. 11.

The simulated evolution of this structure over a very long propagation distance shows that this solitary wave is not a genuine steady-state solution, but it is quite close to being one. It keeps a well-pronounced double-humped shape over, at least, 15 soliton periods. This implies that the double-humped structure is robust enough to be observed in an experiment.

Further increase of b makes the double-humped pulses still less localized, and, eventually, permanent leakage of one of the FF components from the pulse starts. It is difficult to find a critical value of b at which this pulse ceases to exist as a solitary-wave solution, as an extended "tail" of the FF field, the appearance of which signals the onset of the leakage, has a vanishingly small amplitude when it emerges.

5 Conclusion

We have proposed a modification of the usual three-wave second-harmonic-generation model which incorporates two features that are new to the usual model: linear mixing between two components of the fundamental-frequency wave, and a group-velocity mismatch (walkoff) between them. Although the new system is akin to gap-solitons models, its linear spectrum contains no finite gaps. In the temporal domain, the model may be interpreted as the one adding an (effective) twist of the fiber-like waveguide to the birefringence, the latter feature being typical for the Type-II $\chi^{(2)}$ systems. In the spatial domain, the two FF components differ, physically, not by their polarizations, but rather by the orientation of their Poynting vectors in a planar waveguide, the linear coupling being induced by the Bragg grating.

In the absence of the intrinsic walkoff, the linear mixing induces a difference between real soliton solutions with the opposite relative signs between the two FF components, so that they are stable for one sign, and unstable for the other. The development of the instability leads to rearrangement of unstable solitons into stable ones. Adding the intrinsic-walkoff terms, we have found that the evolution leads to formation of stable chirped fundamental solitons, and, additionally, quasi-stable double-humped solitary waves were found.

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Figure Captions

Fig. 1. An example of a stationary real soliton solution with sgn$A$ = sgn$B$. The parameters are $\delta = 2$, $\beta = 0.778$, $\kappa = 0.2$, $\sigma = 3$, $D = 1$, $\alpha = 0.156$, $b_{1,2} = 0$ (no walkoff between the two fundamental-frequency wave components). In this and subsequent figures, the argument x attached to the horizontal axis replaces the variable τ for a case when the model is interpreted in the spatial domain, where x is the transverse coordinate (see the text).

Fig. 2. Evolution of the soliton shown in Fig. 1.

Fig. 3. The distance necessary for the onset of the instability of the soliton with sgn$A$ = sgn$B$ (as detected in the second-harmonic component) vs. the linear-mixing constant κ.

Fig. 4. Evolution of the soliton in the case $D = 0.03$, other parameters taking the same values as in Fig. 1.

Fig. 5. An example of a stationary real soliton with sgn$A = -$sgn$B$ for the same values of parameters as in Fig. 1.

Fig. 6. Evolution of the soliton shown in Fig. 5.

Fig. 7. Evolution of the input pulse identical to the soliton shown in Fig. 5 at the same values of parameters as in Fig. 5, except for $b_1 = -b_2 = 0.1$ (cf. Fig. 6, which pertains to the case $b_{1,2} = 0$).

Fig. 8. Panels (a) and (b) display, respectively, the distribution of local powers, $|A(\tau)|^2$, $|B(\tau)|^2$, and $|C(\tau)|^2$, and local chirps, $(\phi_A)_{\tau\tau}$, $(\phi_B)_{\tau\tau}$, and $(\phi_C)_{\tau\tau}$ (ϕ stands for the phase of field), of the three waves in the (quasi)soliton generated by the evolution process displayed in Fig. 7.

Fig. 9. The net energies of the three components of the soliton, defined as per Eq. (9), vs. the propagation distance z, for the same case as in Fig. 7.

Fig. 10. The total energy (8) of all the three fields vs. z, shown for the same case as in Fig. 9. Note that the energy loss, due to some leakage through the edges of the integration domain, is very small (see numerical values on the vertical axis).

Fig. 11. An example of a quasi-stable double-humped solitary-wave structure found for $\delta = 2$, $\beta = 0.778$, $\kappa = 0.2$, $\sigma = 4$, $D = 1$, $\alpha = 0.156$, and $b_1 = -b_2 = 0.003$. The upper and lower panels show, respectively, the initial configuration at $z = 0$, obtained as a finite-accuracy shooting solution of the stationary equations, and the final configuration obtained at $z = 4000$. As well as in Fig. 8, the distribution of the field powers across the solitary wave is shown here.

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